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Single and double inclusive cross sections for nucleus–nucleus collisions in the perturbative QCD

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Abstract. Single and double inclusive cross sections in nucleus–nucleus collisions are derived in perturbative QCD with interacting BFKL pomerons in the quasi-classical approximation.

1 Introduction

With the advent of colliders the study of particle production in nucleus–nucleus collisions has acquired a prominent role both in experimental and theoretical studies. Single and double inclusive cross sections and correlations related to them naturally draw most attention. In the framework of perturbative QCD they can be studied in the approach based on interacting BFKL pomerons developed for nucleus–nucleus collisions in [1–3]. Intuitive considerations plus experience with the local reggeon field theory (LRFT) allow one to guess the structure of the single inclusive cross section as emission either from the central pomeron or from the two adjacent triple pomeron vertices in the convolution of two sets of fan diagrams propagating from the center to the projectile or target. However, no rigorous demonstration of this structure has been given in the literature. Still worse is the situation with the double inclusive cross section for which even in the LRFT one has a very complicated formula [4].

In this paper we aim at filling this gap. We derive formulas for both single and double inclusive cross sections in nucleus–nucleus collisions in the perturbative QCD approach with interacting BFKL pomerons. Part of the problem, which involves particle emission from the pomerons, is conceptually rather similar to the LRFT (although considerably more complicated technically). So to treat this problem we shall use the cut pomeron formalism developed within the LRFT [5, 6], which we appropriately generalize for the BFKL pomerons. A new problem is emission from the triple pomeron vertex. For the single inclusive case we find that it is indeed described by the simple formulas used in our previous studies [7]. However, the double inclusive cross sections with a single emission from the vertex lead to a complicated expression including different parts of the cut emission vertex found in [8].

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We stress that our derivation is restricted to the leading order in $1/N_c$ and to only tree diagrams for interacting pomerons (classical approximation). Quantum corrections described by loop diagrams have been neglected.

We also note that the problem of single and double inclusive cross sections has been extensively discussed in the past for DIS on the nuclei (with results also applied to proton–nucleus scattering). Breaking of the k_{T} factorization has first been established for single inclusive cross sections in [9] in the dipole formalism with $N_c \to \infty$ and then rederived in [10, 11] in the color glass condensate model and finally in [8] for interacting pomerons. Double inclusive cross sections in DIS on the nucleus were first derived in [12, 13] in the dipole formalism, where it was claimed that they violate the AGK rules. Later in our paper the same cross sections were calculated with interacting pomerons in [8]. It was found there that different cuts of the triple pomeron vertex with parton emission lead to different expressions, which in fact was admitted in the original paper on the AGK rules [14]. However, no contribution was found in which one of the parton was emitted at a higher rapidity than the triple pomeron vertex and the other at a lower rapidity, in violation of the AGK rules. In our opinion the seeming violation observed in [12, 13] may rather be an *artifact* of interpretation of their expressions in terms of pomeron diagrams. We also mention a series of papers devoted to the double inclusive cross sections in DIS on nuclei or proton–nucleus collisions for the drastically simplified case in which the two produced partons have close rapidities and no evolution between them is included [15, 16].

Finally, note that in a recent publication [17] it is claimed that, even in the limit $N_c \to \infty$, in single inclusive cross sections, apart from emission from pomerons and their interaction vertices, one finds emission from higher colorless states made of more than two reggeized gluons (the so-called BKP states) and their interaction vertices. This may be the first place where such states contribute to observable quantities, which may give the possibility of their experimental study. Note, however, that this new contribution vanishes after integration over transverse momenta. So our formulas, which do not take this contribution into account, in any case remain fully valid for distributions in rapidity.

Numerical calculations of the single inclusive cross section in nucleus–nucleus collisions, with vertex emission taken into account, were earlier reported in [7] (as mentioned, without rigorous justification of the formulas). As to the double inclusive cross sections and correlations, due to their complexity, we do not attempt here to use them for numerical studies. These studies present a separate difficult problem and are postponed for future publications.

The paper is organized as follows. In the next section we generalize the cut pomeron formalism for BFKL pomerons. The derivation closely follows that in the LRFT [4–6] with inevitable complications due to the non-local structure of the BFKL pomeron. Next, in Sect. 3 we solve the equation of motion and find working formulas for the single and double inclusive cross section corresponding to emission from the pomeron. In Sect. 4 we discuss emission from the vertex and derive formulas for the single and double inclusive cross sections that involve such emission. Finally, we discuss our results in the last section.

2 Cut pomeron formalism and inclusive cross sections for nucleus–nucleus scattering in the perturbative QCD

2.1 Fields and Lagrangian

Pomerons are described by two fields, ψ and $\psi^\dagger$, which depend on rapidity y, relative gluon momentum q and point b in the transverse space. We shall restrict ourselves to the purely forward case relevant for collisions off large nuclei. Then b is conserved during the collision. Correspondingly we do not indicate it explicitly unless it may lead to confusion. After transition to real fields the generating functional for the Green functions is written as

$$Z = \int D\psi D\psi^\dagger \, \mathrm{e}^{-A} , \tag{1}$$

where the action A is an integral over y, q and b:

$$-A = \int \mathrm{d}y \, \mathrm{d}^2 b \, \mathrm{d}^2 q L(\psi, \psi^\dagger) \tag{2}$$

and the Lagrangian density L is

$$\begin{aligned} L(y,q,b) = {} & \psi^\dagger Q\psi + \lambda\left[(K\psi^\dagger)\psi^2 + (K\psi){\psi^\dagger}^2\right] \\ & + g\delta(y)\psi + f\delta(Y-y)\psi^\dagger . \end{aligned} \tag{3}$$

Here Q is essentially the BFKL operator

$$Q = 2K\left(\frac{\partial}{\partial y} + H\right), \tag{4}$$

with H the BFKL Hamiltonian, and

$$K = \nabla_q^2 q^4 \nabla_q^2 . \tag{5}$$

Operator K is conformal invariant and commutes with H. The triple pomeron coupling λ is

$$\lambda = \frac{4\alpha_s^2 N_c}{\pi} . \tag{6}$$

Finally, $g(y,b)$ and $f(y,b,B)$ represent the coupling of the pomeron to the colliding nuclei. $g(y,b)$ describes this coupling to the target A (at zero rapidity)

$$g(y,b) = AT_A(b)\rho(q) , \tag{7}$$

where $T_A(b)$ is the profile function of nucleus A with its center at the origin in the transverse plane and $\rho(q)$ is the color density of the nucleon. $f(y,b,B)$ describes the coupling to the projectile at rapidity Y and its center at point B (the impact parameter of the collision)

$$f(y,b,B) = BT_B(B-b)\rho(q) . \tag{8}$$

To study different cuts we introduce the cut pomeron formalism following the scheme developed in [5, 6] for the local pomeron. We introduce the six fields

$$\psi_\pm , \quad \psi_c , \quad \psi^\dagger_\pm , \quad \psi^\dagger_c , \tag{9}$$

which describe pomerons to the left (right) of the cut (subindex + (−)) and cut pomerons (subindex c). To fulfill the AGK rules the Lagrangian of these fields is to be taken as

$$L_c = L_c^0 + L_c^I + L_c^E . \tag{10}$$

Here the unperturbed Lagrangian is

$$L_c^0 = \sum_{i=\pm,c} \epsilon_i \psi_i^\dagger Q \psi_i , \tag{11}$$

with

$$\epsilon_\pm = -\epsilon_c = 1 .$$

The interaction involves several terms

$$\begin{aligned} L_c^I = {} & \lambda\left(K\psi_+^\dagger\right)\psi_+^2 + \lambda\left(K\psi_+\right){\psi_+^\dagger}^2 + \lambda\left(K\psi_-^\dagger\right)\psi_-^2 \\ & + \lambda\left(K\psi_-\right){\psi_-^\dagger}^2 - 2\lambda\left(K\psi_c^\dagger\right)\psi_c\left(\psi_+ + \psi_-\right) \\ & - 2\lambda\left(K\psi_c\right)\psi_c^\dagger\left(\psi_+^\dagger + \psi_-^\dagger\right) \\ & + \sqrt{2}\lambda\left(K\psi_c^\dagger\right)\left(\psi_+\psi_- + \psi_c^2\right) \\ & + \sqrt{2}\lambda(K\psi_c)(\psi_+^\dagger\psi_-^\dagger + {\psi_c^\dagger}^2) . \end{aligned} \tag{12}$$

Finally, the external Lagrangian is

$$\begin{aligned} L_c^E = {} & g\delta(y)\left(\psi_+ + \psi_- - \sqrt{2}\psi_c\right) \\ & + f\delta(Y-y)\left(\psi_+^\dagger + \psi_-^\dagger - \sqrt{2}\psi_c^\dagger\right) . \end{aligned} \tag{13}$$

2.2 Inclusive cross sections

To study inclusive cross sections we additionally introduce the interaction of the cut pomeron with the emitted particles. As is well known, the inclusive cross section corresponding to emission of a gluon jet from the pomeron can be written as a double integral:

$$I(y,\kappa) \equiv \frac{(2\pi)^3\,\mathrm{d}\sigma}{\mathrm{d}y\,\mathrm{d}^2\kappa} = 2\int \mathrm{d}^2b\,\mathrm{d}^2q_1\,\mathrm{d}^2q_2\delta^2(q_1+q_2-\kappa) \times \eta(q_1,q_2)p_A(y,q_1,b)p_B(y,q_2,b)\,, \tag{14}$$

where $p_{A(B)}(y,q,b)$ is the pomeron coupled to the target (projectile) and

$$\eta(q_1,q_2) = \frac{16\pi^2\alpha_{\mathrm{s}}N_c}{\kappa^2}q_1^2\nabla_1^2q_2^2\nabla_2^2\,. \tag{15}$$

This expression can be conveniently rewritten in the coordinate space in the local form, in terms of the 'non-amputated' pomeron

$$P(y,r,b) = 2\pi r^2\int\frac{\mathrm{d}^2q}{2\pi}\,\mathrm{e}^{\mathrm{i}qr}p(y,q,b) \tag{16}$$

as

$$I(y,\kappa) = \frac{8\alpha_{\mathrm{s}}N_c}{\kappa^2}\int \mathrm{d}^2b\,\mathrm{d}^2rP_A(y,r,b)\nabla^2\,\mathrm{e}^{\mathrm{i}\kappa r}\nabla^2P_B(y,r,b)\,. \tag{17}$$

To generate inclusive cross sections we add a term to the action

$$-\Delta A_{\mathrm{c}} = \int \mathrm{d}y\,\mathrm{d}^2b\,\mathrm{d}^2q_1\,\mathrm{d}^2q_2\xi(y,q_1,q_2,b)\psi_{\mathrm{c}}^\dagger(y,q_1,b) \times \psi_{\mathrm{c}}(y,q_2,b)\,. \tag{18}$$

Differentiation by ξ then gives insertions into the cut pomeron propagator at rapidity y and transverse point b with momenta q_1 and q_2. As in [4] we denote the total action by

$$A_{\mathrm{c}}(\xi) = A_{\mathrm{c}} + \Delta A_{\mathrm{c}}\,. \tag{19}$$

It follows from unitarity that

$$A_{\mathrm{c}}(\xi=0) = 0\,. \tag{20}$$

Also one finds that the generating functional of the amplitudes at $\xi\neq 0$ is given by

$$T_{\mathrm{c}}(\xi) = 1-S+\frac{1}{2}\left(\mathrm{e}^{-A_{\mathrm{c}}(\xi)}-1\right) \tag{21}$$

where S is just the S matrix at $\xi=0$ [4] (also see Appendix A).

The single and double inclusive cross sections are obtained as

$$I_1(y,\kappa) = 2\int \mathrm{d}^2b\,\mathrm{d}^2q_1\,\mathrm{d}^2q_2\delta^2(q_1+q_2-\kappa)\eta(q_1,q_2) \times \left.\frac{\delta T_{\mathrm{c}}(\xi)}{\delta\xi(y,q_1,q_2,b)}\right|_{\xi=0} \tag{22}$$

and

$$I_2(y,\kappa|y',\kappa') = 2\int \mathrm{d}^2b\,\mathrm{d}^2b'\,\mathrm{d}^2q_1\,\mathrm{d}^2q_2\delta^2(q_1+q_2-\kappa) \times \mathrm{d}^2q_1'\,\mathrm{d}^2q_2'\delta^2(q_1'+q_2'-\kappa')\eta(q_1,q_2)\eta(q_1',q_2') \times \left.\frac{\delta^2T_{\mathrm{c}}(\xi)}{\delta\xi(y,q_1,q_2,b)\delta\xi(y',q_1'q_2',b')}\right|_{\xi=0}\,. \tag{23}$$

Next we use the property proven in [4] that at any values of ξ, due to the equations of motion,

$$-\frac{\delta A_{\mathrm{c}}(\xi)}{\delta\xi(y,q_1,q_2,b)} = \psi_{\mathrm{c}}^\dagger(y,q_1,b,\xi)\psi_{\mathrm{c}}(y,q_2,b,\xi)\,. \tag{24}$$

This together with (20) immediately gives a simple formula for the single inclusive cross section:

$$I_1(y,q) = \int \mathrm{d}^2b\,\mathrm{d}^2q_1\,\mathrm{d}^2q_2\delta^2(q_1+q_2-q)\eta(q_1,q_2) \times \psi_{\mathrm{c}}^\dagger(y,q_1,b,\xi=0)\psi_{\mathrm{c}}(y,q_2,b,\xi=0)\,. \tag{25}$$

The formula for the calculation of the double functional derivative at $\xi=0$ becomes

$$\left.\frac{\delta^2T_{\mathrm{c}}(\xi)}{\delta\xi(y,q_1,q_2,b)\delta\xi(y',q_1'q_2',b')}\right|_{\xi=0} = \frac{1}{2}\Big[\psi_{\mathrm{c}}^\dagger(y,q_1,b,\xi)\psi_{\mathrm{c}}(y,q_2,b,\xi)\psi_{\mathrm{c}}^\dagger(y',q_1',b',\xi) \times \psi_{\mathrm{c}}(y',q_2',b',\xi) + \frac{\delta}{\delta\xi(y,q_1,q_2,b)}\Big(\psi_{\mathrm{c}}^\dagger(y',q_1',b',\xi)\psi_{\mathrm{c}}(y',q_2',b',\xi)\Big)\Big]_{\xi=0}\,. \tag{26}$$

So the whole problem is reduced to finding ψ_{c}, $\psi_{\mathrm{c}}^\dagger$ and their first derivatives in ξ at $\xi=0$.

2.3 Unitary transformation to new variables and equations of motion

Considerable simplification can be achieved if, following [4], we make a unitary transformation of our fields introducing the new fields $\phi_\pm$ and ϕ_0, and the conjugate fields $\pi_\pm$ and π_0 by

$$\psi_\pm = \phi_\pm,\quad \psi_{\mathrm{c}} = -\frac{1}{\sqrt{2}}(\phi_0-\phi_+-\phi_-)\,,\quad \psi_\pm^\dagger = \pi_\pm+\pi_0,\quad \psi_{\mathrm{c}}^\dagger = \sqrt{2}\pi_0. \tag{27}$$

This transforms L_{c} into a new Lagrangian:

$$L_{\mathrm{c}}' = \sum_{i=+,-,0}\pi_iQ\phi_i + \lambda\sum_{i=+,-,0}\left((K\pi_i)\phi_i^2+(K\phi_i)\pi_i^2\right) + \lambda\Big(\pi_+\pi_0K(\phi_0+\phi_+-\phi_-)+\pi_-\pi_0K(\phi_0-\phi_++\phi_-)) + \pi_+\pi_-K(\phi_++\phi_--\phi_0)\Big)\,. \tag{28}$$

The coupling to the nuclei takes the form

$$L_E' = g\delta(y)\phi_0 + f\delta(y-Y)(\pi_++\pi_-) \tag{29}$$

and the part of the action depending on ξ becomes

$$-\varDelta A_{\mathrm{c}} = \int \mathrm{d}y\,\mathrm{d}^2q_1\,\mathrm{d}^2q_2\xi(y,q_1,q_2)\pi_0(q_1) \times \Big(\phi_+(q_2)+\phi_-(q_2)-\phi_0(q_2)\Big). \tag{30}$$

Variation of the action with respect to our fields gives the equations of motion. It can be seen that they are totally identical for the $\pm$ fields. So it is sufficient to write them down for $\phi=\phi_+=\phi_-$ and ϕ_0, and $\pi=\pi_+=\pi_-$ and π_0. They are (at point y,q):

$$\pi Q + 2\lambda K(\pi^2) + 2\lambda\phi K\pi + \int \mathrm{d}^2k\xi(y,q,k)\pi_0(k) = 0\,, \tag{31}$$

$$\pi_0 Q + \lambda K(\pi_0^2 - \pi^2 + 2\pi_0\pi) + 2\lambda\phi_0 K\pi - \int \mathrm{d}^2k\xi(y,q,k)\pi_0(k) + g\delta(y) = 0\,, \tag{32}$$

$$Q\phi + \lambda K\phi^2 + \lambda\pi K(4\phi-\phi_0) + \lambda\pi_0 K\phi_0 + f\delta(Y-y) = 0\,, \tag{33}$$

$$Q\phi_0 + \lambda K\phi_0^2 + 2\lambda(\pi_0+\pi)K\phi_0 + \int \mathrm{d}^2k\xi(y,q,k)\Big(2\phi(k)-\phi_0(k)\Big). \tag{34}$$

3 Inclusive cross sections due to emission from the pomeron

3.1 Solution of the equation of motion: fields at $\xi = 0$ and single inclusive cross sections

Our first task is to find the solutions of our equations of motion at $\xi=0$, which knowledge is sufficient for the single inclusive cross section according to (25).

If $\xi=0$ then the equations for π and ϕ_0 do not contain driving terms, so that these fields are identically zero:

$$\pi(y,q) = \phi_0(y,q) = 0\,. \tag{35}$$

The two remaining equations decouple:

$$\pi_0 Q + \lambda K(\pi_0^2) + g\delta(y) = 0 \tag{36}$$

and

$$Q\phi + \lambda K\phi^2 + f\delta(Y-y) = 0\,. \tag{37}$$

These are the standard BK equations for the sum of fan diagrams χ that go to the point (y,q) from the target:

$$\pi_0(y,q) = \chi(y,q,g) \tag{38}$$

or from the projectile:

$$\phi(y,q) = \chi(Y-y,q,f) \equiv \tilde{\chi}\,. \tag{39}$$

Put into the expression (25) they give the commonly used factorized expression for the single inclusive cross section as a convolution of two sets of fan diagrams:

$$I_1(y,\kappa) = 2\int \mathrm{d}^2b\,\mathrm{d}^2q_1\,\mathrm{d}^2q_2\delta^2(q_1+q_2-\kappa)\eta(q_1,q_2) \times \chi(y,q_1,b)\tilde{\chi}(y,q_2,b)\,; \tag{40}$$

or, in the coordinate space, in terms of $Z(y,r,b)$ related to $\chi(y,q,b)$ similarly to (16):

$$Z(y,r,b) = 2\pi r^2 \int \frac{\mathrm{d}^2q}{2\pi}\,\mathrm{e}^{\mathrm{i}qr}\chi(y,q,b)\,, \tag{41}$$

as

$$I(y,\kappa) = \frac{8\alpha_{\mathrm{s}}N_c}{\kappa^2}\int \mathrm{d}^2b\,\mathrm{d}^2r Z(y,r,b)\nabla^2\,\mathrm{e}^{\mathrm{i}\kappa r}\nabla^2\tilde{Z}(y,r,b)\,. \tag{42}$$

3.2 Derivative of the fields in ξ at $\xi = 0$ and double inclusive cross sections

To set up equations for the derivatives of the fields in ξ at $\xi=0$ we have to differentiate (31)–(34) in $\xi(y_1,q_1,q_2,b_1)$ and then put $\xi=0$. The field derivatives and the equations as a whole will depend on seven variables: y,q,b and y_1,q_1,q_2,b_1, which we shall show explicitly only when it is necessary. Thus $\delta\pi/\delta\xi$, say, will in fact mean $\delta\pi(y,q,b)/\delta\xi(y_1,q_1,q_2,b_1)$. With this notation we get the equations

$$\Big(-\frac{\partial}{\partial y} + H + \lambda\tilde{\chi}\Big)2K\frac{\delta\pi}{\delta\xi} + \delta(y-y_1)\delta^2(b-b_1)\delta^2(q-q_1)\chi(q_2) = 0\,, \tag{43}$$

$$2K\Big(-\frac{\partial}{\partial y} + H + \lambda\chi\Big)\frac{\delta\pi}{\delta\xi} + 2\lambda\frac{\delta\phi_0}{\delta\xi}K\chi + 2\lambda K\Big(\chi\frac{\delta\pi}{\delta\xi}\Big) - \delta(y-y_1)\delta^2(b-b_1)\delta^2(q-q_1)\chi(q_2) = 0\,, \tag{44}$$

$$2K\Big(\frac{\partial}{\partial y} + H + \lambda\tilde{\chi}\Big)\frac{\delta\phi}{\delta\xi} + 4\lambda\frac{\delta\pi}{\delta\xi}K\tilde{\chi} + \lambda\chi K\frac{\delta\phi_0}{\delta\xi} = 0\,, \tag{45}$$

$$\Big(\frac{\partial}{\partial y} + H + \lambda\chi\Big)2K\frac{\delta\phi_0}{\delta\xi} + 2\delta(y-y_1)\delta^2(b-b_1)\delta^2(q-q_1)\tilde{\chi}(q_2) = 0\,. \tag{46}$$

To simplify these multivariable equations we first note that the b dependence of the derivatives is trivial: obviously they all are proportional to $\delta^2(b-b_1)$. So we separate this factor and consider the derivatives at a fixed point b, which need not be shown explicitly. Next we return to our expression for the double inclusive cross section. Obviously it consists of two terms. One is just the product of two single inclusive cross sections:

$$I_2^{(1)}(y_1,\kappa_1|y_2,\kappa_2) = I_1(y_1,\kappa_1)I(y_2,\kappa_2). \tag{47}$$

The other term comes from the second term in (26) and has the form

$$
\begin{aligned}
I_2^{(2)}(y,\kappa|y',\kappa') = & \int \mathrm{d}^2b\,\mathrm{d}^2b'\,\mathrm{d}^2q_1\,\mathrm{d}^2q_2\delta^2(q_1+q_2-\kappa) \\
& \times \mathrm{d}^2q_1'\,\mathrm{d}^2q_2'\delta^2(q_1'+q_2'-\kappa')\eta(q_1,q_2)\eta(q_1',q_2') \\
& \times \frac{\delta}{\delta\xi(y,q_1,q_2,b)}\psi_\mathrm{c}^\dagger(y',q_1',b',\xi)\psi_\mathrm{c}(y',q_2',b'\xi)\Big|_{\xi=0} \\
= & \int \mathrm{d}^2q_1\,\mathrm{d}^2q_2\delta^2(q_1+q_2-\kappa)\,\mathrm{d}^2q_1'\,\mathrm{d}^2q_2' \\
& \times \delta^2(q_1'+q_2'-\kappa')\eta(q_1,q_2)\eta(q_1',q_2') \\
& \times \Bigg(\psi_\mathrm{c}^\dagger(y',q_1',b',\xi=0)\;\frac{\delta\psi_\mathrm{c}(y',q_2',b',\xi)}{\delta\xi(y,q_1,q_2,b)}\Big|_{\xi=0} \\
& + \psi_\mathrm{c}(y',q_1',b',\xi=0)\;\frac{\delta\psi_\mathrm{c}^\dagger(y',q_2'b',\xi)}{\delta\xi(y,q_1,q_2b)}\Big|_{\xi=0}\Bigg). \qquad (48)
\end{aligned}
$$

Due to the factor $\delta^2(b-b')$ contained in the derivatives the double integrations in b and b' in fact turn into one over the common point b. Taking this into account and suppressing this common argument b, in terms of the fields ϕ and π we find

$$
\begin{aligned}
I_2^{(2)}(y,\kappa|y',\kappa') = & \int \mathrm{d}^2b\,\mathrm{d}^2q_1\,\mathrm{d}^2q_2\delta^2(q_1+q_2-\kappa)\,\mathrm{d}^2q_1'\,\mathrm{d}^2q_2' \\
& \times \delta^2(q_1'+q_2'-\kappa')\eta(q_1,q_2)\eta(q_1',q_2') \\
& \times \Bigg(\chi(y',q_1')\;\frac{\delta(2\phi(y',q_2',\xi)-\phi_0(y',q_2',\xi))}{\delta\xi(y,q_1,q_2)}\Big|_{\xi=0} \\
& + 2\tilde\chi(y',q_1')\;\frac{\delta\pi_0(y',q_2',\xi)}{\delta\xi(y,q_1,q_2)}\Big|_{\xi=0}\Bigg), \qquad (49)
\end{aligned}
$$

where it is assumed that the factor $\delta^2(b-b')$ has been dropped from the derivatives. From this formula we can conclude that we do not need our derivatives in $\xi(y_1,q_1,q_2)$ at all values of its arguments but rather integrated over q_1 and q_2 with weight $\delta(q_1+q_2-\kappa)\eta(q_1,q_2)$. Correspondingly, we define (suppressing the argument b)

$$
\Pi_0(y,q,y_1,\kappa) = \int \mathrm{d}^2q_1\,\mathrm{d}^2q^2\delta^2(q_1+q_2-\kappa)\eta(q_1,q_2) \times \frac{\delta\pi_0(y,q,\xi)}{\delta\xi(y_1,q_1,q_2)}\Big|_{\xi=0}, \qquad (50)
$$

$$
\Pi(y,q,y_1,\kappa) = \int \mathrm{d}^2q_1\,\mathrm{d}^2q^2\delta^2(q_1+q_2-\kappa)\eta(q_1,q_2) \times \frac{\delta\pi(y,q,\xi)}{\delta\xi(y_1,q_1,q_2)}\Big|_{\xi=0}, \qquad (51)
$$

$$
\Phi_0(y,q,y_1,\kappa) = \int \mathrm{d}^2q_1\,\mathrm{d}^2q^2\delta^2(q_1+q_2-\kappa)\eta(q_1,q_2) \times \frac{\delta\phi_0(y,q,\xi)}{\delta\xi(y_1,q_1,q_2)}\Big|_{\xi=0}, \qquad (52)
$$

$$
\Phi(y,q,y_1,\kappa) = \int \mathrm{d}^2q_1\,\mathrm{d}^2q^2\delta^2(q_1+q_2-\kappa)\eta(q_1,q_2) \times \frac{\delta\phi(y,q,\xi)}{\delta\xi(y_1,q_1,q_2)}\Big|_{\xi=0}. \qquad (53)
$$

In view of expression (49) we also introduce

$$
\Psi(y,q,y_1,\kappa) = 2\Phi(y,q,y_1,\kappa) - \Phi_0(y,q,y_1,\kappa). \qquad (54)
$$

In terms of these quantities we find

$$
\begin{aligned}
& I_2^{(2)}(y,\kappa|y',\kappa') \\
& = \int \mathrm{d}^2b\,\mathrm{d}^2q_1'\,\mathrm{d}^2q_2'\delta^2(q_1'+q_2'-\kappa')\eta(q_1',q_2') \\
& \times \Big(\chi(y',q_1')\Psi(y',q_2',y,\kappa) + 2\tilde\chi(y',q_1')\Pi_0(y',q_2',y,\kappa)\Big). \qquad (55)
\end{aligned}
$$

Integrating (43)–(46) over q_1 and q_2 with weight $\delta^2(q_1+q_2-\kappa)\eta(q_1,q_2)$ we get the following system:

$$
\begin{aligned}
& \left(-\frac{\partial}{\partial y}+H+\lambda\tilde\chi\right)2K\Pi \\
& \quad \times +\delta(y-y_1)\eta(q,\kappa-q)\chi(y,\kappa-q) = 0, \qquad (56) \\
& 2K\Big(-\frac{\partial}{\partial y}+H+\lambda\chi\Big)\Pi_0 + 2\lambda\Phi_0K\chi + 2\lambda K(\chi\Pi) \\
& \quad - \delta(y-y_1)\eta(q,\kappa-q)\chi(y,\kappa-q) = 0, \qquad (57) \\
& 2K\Big(\frac{\partial}{\partial y}+H+\lambda\tilde\chi\Big)\Psi + 8\lambda\Pi K\tilde\chi + 2\lambda K(\tilde\chi\Phi_0) \\
& \quad - 2\delta(y-y_1)\eta(q,\kappa-q)\tilde\chi(y,\kappa-q) = 0, \qquad (58) \\
& \left(\frac{\partial}{\partial y}+H+\lambda\chi\right)2K\Phi_0 \\
& \quad + 2\delta(y-y_1)\eta(q,\kappa-q)\tilde\chi(y,\kappa-q) = 0. \qquad (59)
\end{aligned}
$$

One observes that the first and last equations determine Π and Φ_0 in terms of the known χ and $\tilde\chi$, after which the second and third equation allow us to find Π_0 and Ψ.

Equations (56)–(59) are convenient for numerical calculations. However, one can also formally express the solution in terms of Green function of the operators $\partial/\partial y + H + \lambda\chi$ and $\partial/\partial y + H + \tilde\chi$ to obtain formulas that can be compared with [4].

3.3 Formal solution

We define the Green functions:

$$
\begin{aligned}
G &= \Big(\frac{\partial}{\partial y}+H+\lambda\chi\Big)^{-1}, \quad G^\mathrm{T} = \Big(-\frac{\partial}{\partial y}+H+\lambda\chi\Big)^{-1}, \\
\tilde G &= \Big(\frac{\partial}{\partial y}+H+\lambda\tilde\chi\Big)^{-1}, \quad \tilde G^\mathrm{T} = \Big(-\frac{\partial}{\partial y}+H+\lambda\tilde\chi\Big)^{-1}. \qquad (60)
\end{aligned}
$$

Each Green function is an integral operator in (y,q) space, so that e.g. for G the kernel is $G(y,q|y',q')$. In our formulas the rapidities and momenta enter asymmetrically, so in many cases we shall suppress the momenta but leave the rapidities, considering $G_{yy'}$ as an operator in the momentum space with the kernel $G_{yy'}(q|q')$.

In this notation, the solution of (56) and (59) is immediate:

$$
\Pi(y) = -\frac{1}{2}K^{-1}\tilde G^\mathrm{T}_{y,y_1}(\eta\chi)_{y_1}, \qquad (61)
$$

$$\Phi_0(y) = -K^{-1}G_{y,y_1}(\eta\tilde{\chi})_{y_1}\,. \qquad (62)$$

We put these solutions into (57) multiplied by $(2K)^{-1}$ to obtain

$$\left(-\frac{\partial}{\partial y}+H+\lambda\chi\right)\Pi_0-\lambda K^{-1}(K\chi(y))K^{-1}G_{y,y_1}(\eta\tilde{\chi})_{y_1} -\frac{1}{2}\chi K^{-1}\tilde{G}^{\mathrm{T}}_{y,y_1}(\eta\chi)_{y_1}-\delta(y-y_1)\frac{1}{2}K^{-1}(\eta\chi_{y_1})=0\,. \quad (63)$$

Applying operator G^{T} we get

$$\Pi_0=\lambda\left[G^{\mathrm{T}}K^{-1}(K\chi)K^{-1}G\right]_{yy_1}(\eta\tilde{\chi})_{y_1} +\frac{1}{2}\lambda\left[G^{\mathrm{T}}\chi K^{-1}\tilde{G}^{\mathrm{T}}\right]_{y,y_1}(\eta\chi)_{y_1}+\frac{1}{2}G^{\mathrm{T}}_{yy_1}K^{-1}(\eta\chi)_{y_1}\,. \quad (64)$$

Similar operations with (58) first give the equation

$$\left(\frac{\partial}{\partial y}+H+\lambda\tilde{\chi}\right)\Psi-2\lambda K^{-1}(K\tilde{\chi})K^{-1}\tilde{G}^{\mathrm{T}}_{y,y_1}(\eta\chi)_{y_1} -\lambda\tilde{\chi}K^{-1}G_{y,y_1}(\eta\tilde{\chi})_{y_1}-\delta(y-y_1)K^{-1}\eta\tilde{\chi}_{y_1}=0\,, \quad (65)$$

which after application of the operator $\tilde{G}$ gives

$$\Psi=2\lambda\Big[\tilde{G}K^{-1}(K\tilde{\chi})K^{-1}\tilde{G}^{\mathrm{T}}\Big]_{y,y_1}(\eta\chi)_{y_1} +\lambda\Big[\tilde{G}\tilde{\chi}K^{-1}G\Big]_{y,y_1}(\eta\tilde{\chi})_{y_1}+\tilde{G}_{yy_1}K^{-1}\eta\tilde{\chi}_{y_1}\,. \quad (66)$$

One observes that Ψ is obtained from $2\Pi_0$ by the substitutions

$$G\to\tilde{G}^{\mathrm{T}},\tilde{G}\to G^{\mathrm{T}},\ \chi\leftrightarrow\tilde{\chi}\,. \qquad (67)$$

Putting the obtained expressions for the field derivatives (61), (62), (64) and (66) into (55) and using the property (67) we obtain the part $I_2^{(2)}$ of the inclusive cross section:

$$\begin{aligned} &I_2^{(2)}(y,\kappa|y',\kappa') \\ &=\int \mathrm{d}^2b\,\mathrm{d}^2q_1'\,\mathrm{d}^2q_2'\delta^2(q_1'+q_2'-\kappa')\eta(q_1',q_2')\tilde{\chi}_{y',q_1'} \\ &\times\Big\{2\lambda\Big[G^{\mathrm{T}}K^{-1}(K\chi)K^{-1}G\Big]_{y',q_2'|y,q_2}(\eta\tilde{\chi})_{y,q_2} \\ &+\lambda\Big[G^{\mathrm{T}}\chi K^{-1}\tilde{G}^{\mathrm{T}}\Big]_{y',q_2'|,y,q_2}(\eta\chi)_{y,q_2} \\ &+G^{\mathrm{T}}_{y',q_2'|y,q_2}K^{-1}(\eta\chi)_{y,q_2}\Big\} \\ &+\Big(G\to\tilde{G}^{\mathrm{T}},\tilde{G}\to G^{\mathrm{T}},\chi\leftrightarrow\tilde{\chi}\Big). \end{aligned} \qquad (68)$$

The obtained expression (68) is similar to the one in the framework of LRFT [4], except that in our non-local case all quantities are operators also in the momentum space and that in various places there appear operators K and K^{-1} acting in this space. Its diagrammatic illustration is presented in Fig. 1. In this figure external lines with crosses (circles) show sums of fan diagrams propagating towards the target χ (projectile $\tilde{\chi}$). Correspondingly, the internal lines marked with crosses (circles) show the Green functions G ($\tilde{G}$). Horizontal lines indicate the two observed particles. To the diagrams shown in Fig. 1 one should add similar ones with the target and projectile interchanged.

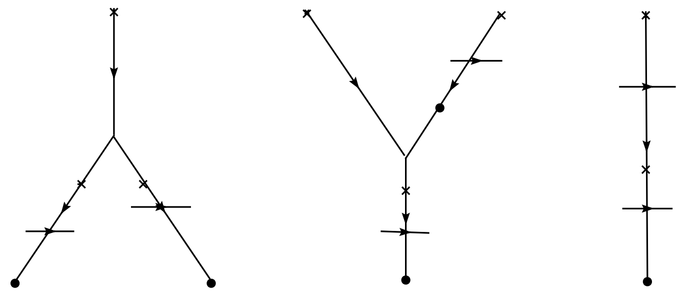

Fig. 1. Diagrams corresponding to the non-trivial part $I_2^{(2)}$ of the double emission from pomerons. Crosses mark χ and G, full circles mark $\tilde{\chi}$ and $\tilde{G}$

From the practical point of view expression, (68) is not very useful due to multiple integrations in (y,q) space. Numerical solution of the evolution equations (56)–(59) seems more promising.

4 Emission from the vertex

To include emission from the triple pomeron vertex we have to add new parts to the Lagrangian, which describe this emission. In the splitting vertex the pomeron before the split always has to be cut. The two emerging pomerons may be both cut and uncut. In accordance with the structure of the interaction in (12) the vertex emission part of the action is to be taken as

$$-A_\gamma=\int \mathrm{d}y\,\mathrm{d}^2b\,\mathrm{d}^2q_1\,\mathrm{d}^2q_2\,\mathrm{d}^2q_3L_\gamma+\mathrm{h.c.}\,, \qquad (69)$$

where

$$\begin{aligned} L_\gamma=\sqrt{2}\psi_{\mathrm{c}}^\dagger(q_1)\Big(&\gamma_d(q_1|q_2,q_3)\psi_+(q_2)\psi_-(q_3) \\ &-\sqrt{2}\gamma(q_1|q_2,q_3)\psi_{\mathrm{c}}(q_2)(\psi_+(q_3)+\psi_-(q_3)) \\ &+\gamma_{\mathrm{c}}(q_1|q_2,q_3)\psi_{\mathrm{c}}(q_2)\psi_{\mathrm{c}}(q_3)\Big). \end{aligned} \qquad (70)$$

The part explicitly shown corresponds to emission from the splitting vertex. The part corresponding to the emission from the merging vertex is indicated as h.c. The vertex functions γ_d, γ and γ_{c} describe emissions from the diffractive, single and double cuts of the triple pomeron vertex, respectively. They are real functions of the three relative momenta of the joining pomerons and carry a factor $\delta^2(\kappa+q_1-q_2-q_3)$, where κ is the momentum of the emitted jet. They are different and their form has been found in [1] and is reproduced in Appendix B. The fields in (70) are supposed to be taken at the same rapidity y and transverse point b, which dependence is not shown explicitly. In the perturbation expansion the new interaction term A_γ has to be taken the number of times that corresponds to the number of emissions from the vertex. So for the single inclusive cross section we have to take it only once and for the double inclusive cross section at most twice.

The single inclusive cross section corresponding to the emission from the vertex is obtained from (70) just by substituting the fields by solutions of the equations of motion (31)–(34) at $\xi=0$ and integrating over b. For the fields ψ and $\psi^\dagger$ these solutions are

$$\psi_\pm=\tilde{\chi},\ \psi_{\rm c}=\sqrt{2}\tilde{\chi},\ \psi^\dagger_\pm=\chi,\ \psi^\dagger_{\rm c}=\sqrt{2}\chi\,. \tag{71}$$

Thus we find a contribution

$$\begin{aligned} I_1^{(\gamma)}(y,\kappa)=2\int {\rm d}^2b\prod_{i=1}^{3}{\rm d}^2q_i\Big(&\gamma_d(q_1|q_2,q_3)-4\gamma(q_1|q_2,q_3)\\ &+2\gamma_{\rm c}(q_1|q_2,q_3)\Big)\\ &\times\Big(\chi(q_1)\tilde{\chi}(q_2)\tilde{\chi}(q_3)+\tilde{\chi}(q_1)\chi(q_2)\chi(q_3)\Big). \end{aligned} \tag{72}$$

The total vertex

$$\gamma^{\rm tot}=\gamma_d-4\gamma+2\gamma_{\rm c} \tag{73}$$

has a simple form in the coordinate space. If we introduce the coordinate vertex $\Gamma(r_1|r_2,r_3)$ acting on non-amputated pomerons according to the relation

$$\begin{aligned} &\int {\rm d}^2q_1\,{\rm d}^2q_2\,{\rm d}^2q_3\chi(q_1)\gamma(q_1|q_2,q_3)\tilde{\chi}(q_2)\tilde{\chi}(q_3)\\ &=\int {\rm d}^2r_1\,{\rm d}^2r_2\,{\rm d}^2r_3 Z(r_1)\Gamma(r_1|r_2,r_3)\tilde{Z}(r_2)\tilde{Z}(r_3), \end{aligned} \tag{74}$$

where $\chi(q)$ and $Z(r)$ are related by (41), then one finds [8]

$$\Gamma^{\rm tot}(r_1|r_2,r_2)=-\frac{2\alpha_{\rm s}N_c}{\kappa^2}\nabla_1^2{\rm e}^{{\rm i}\kappa r_1}\nabla_1^2\delta^2(r_2-r_1)\delta^2(r_3-r_1). \tag{75}$$

It corresponds to the expression first obtained in [9] as a contribution additional to the emission from the pomeron.

Passing to the double inclusive cross section we first find a contribution corresponding to the emission from two vertices, which is obtained by taking a product of two interactions (70) with different external momenta κ and κ', at different rapidities y and y' and transverse points b and b', substituting in them the fields by the solution of the equations of motion and integrating over both transverse points b and b'. As a result we obviously find a product of two single inclusive cross sections (72):

$$I_2^{(\gamma\gamma)}(y,\kappa|y',\kappa')=I_1^{(\gamma)}(y,\kappa)I_1^{(\gamma)}(y',\kappa')\,. \tag{76}$$

To find the mixed contribution in which one jet is emitted from the vertex and the other from the pomeron we have to consider the theory with the interaction term $\Delta A_{\rm c}$, (18), and once differentiate in ξ:

$$\begin{aligned} &I_2^{(\gamma)}(y,\kappa|y'\kappa')\\ &=\int {\rm d}^2b\,{\rm d}^2b'\prod_{i=1}^{3}{\rm d}^2q_i\,{\rm d}^2q'_1\,{\rm d}^2q'_2\delta^2(q'_1+q'_2-\kappa')\eta(q'_1.q'_2)\\ &\times\Big[\frac{\delta}{\delta\xi(y',q'_1,q'_2,b')}L_\gamma(y,b,\kappa,q_1,q_2,q_3,\xi)\,{\rm e}^{-A_{\rm c}(\xi)}\Big]_{\xi=0}\\ &+\Big(y\leftrightarrow y',\kappa\leftrightarrow\kappa'\Big). \end{aligned} \tag{77}$$

Differentiation in ξ will give two terms. One comes from the differentiation of the exponential. After integrations over b' and q'_1 and q'_2 with weight η, this differentiation will give the single inclusive cross section $I_1(y',\kappa')$ corresponding to emission from the pomeron. The factor L_γ at $\xi=0$ will generate the single inclusive cross section (72). As a result this part gives a contribution

$$I_1^{(\gamma)}(y,\kappa)I_1(y',\kappa')+I_1^{(\gamma)}(y',\kappa')I_1(y,\kappa)\,. \tag{78}$$

If we introduce the total single inclusive cross section

$$I_1^{\rm tot}(y,\kappa)=I_1(y,\kappa)+I^{(\gamma)}(y,\kappa)\,, \tag{79}$$

then collecting (47), (76) and (78) we find a factorized contribution to the double inclusive cross section

$$I_2^{\rm fact}(y,\kappa|y'\kappa')=I^{\rm tot}(y,\kappa)I^{\rm tot}(y'\kappa')\,. \tag{80}$$

The second part of $I_2^{(\gamma)}$ will come from the differentiation in ξ of the fields inside L_γ, the exponential factor giving unity. Differentiation in ξ together with integrations over q'_1 and q'_2 with weight η will substitute the fields in accordance with (50)–(54). If we define

$$D\equiv\int {\rm d}^2q_1\,{\rm d}^2q_2\delta^2(q_1+q_2-\kappa)\eta(q_1,q_2)\;\frac{\delta}{\delta\xi(y_1,q_1,q_2}\bigg|_{\xi=0}\,, \tag{81}$$

then in terms of fields ψ and $\psi^\dagger$ we find

$$\begin{aligned} D\psi_{\rm c}&=\frac{1}{\sqrt{2}}\Psi\,,\quad D\psi_\pm=\Phi\,,\\ D\psi^\dagger_{\rm c}&=\sqrt{2}\Pi_0\,,\quad D\psi^\dagger_\pm=\Pi+\Pi_0\,. \end{aligned} \tag{82}$$

We recall that the derivatives are proportional to $\delta^2(b-b')$, so that the double integration over b and b' turns into a single one. We then obtain the following expression for the non-factorized part of the double inclusive cross section $I_2^{(\gamma)}$:

$$\begin{aligned} &I_2^{(\gamma,nf)}(y,\kappa|y'\kappa')\\ &=\int {\rm d}^2b\prod_{i=1}^{3}{\rm d}^2q_i\Big\{2\gamma^{\rm tot}(q_1|q_1,q_3)\Pi_0(q_1)\tilde{\chi}(q_2)\tilde{\chi}(q_3)\\ &+2\chi(q_1)\tilde{\chi}(q_3)\Big[\gamma^{\rm tot}(q_1|q_2,q_3)\Psi(q_2)\\ &+\Big(\gamma_d(q_1|q_2,q_3)-2\gamma(q_1|q_2,q_3)\Big)\Phi_0(q_2)\Big]\\ &+\gamma^{\rm tot}(q_1|q_1,q_3)\Psi(q_1)\chi(q_2)\chi(q_3) \end{aligned}$$

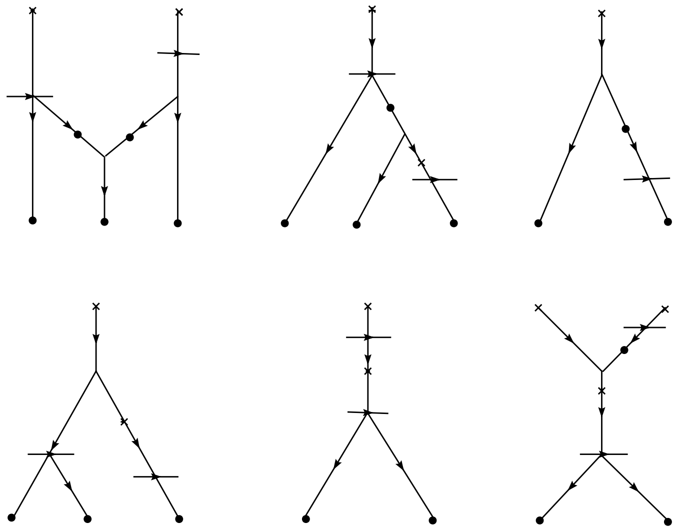

Fig. 2. Diagrams for the non-factorized part $I_2^{(\gamma,nf)}$ of the double emission from a pomeron and a vertex. Notations are as in Fig. 1

$$
\begin{aligned}
&+4\tilde{\chi}(q_1)\chi(q_3)\Big[\gamma^{\mathrm{tot}}(q_1|q_2,q_3)\Pi_0(q_2)\\
&+\Big(\gamma_d(q_1|q_2,q_3)-2\gamma(q_1|q_2,q_3)\Big)\Pi(q_2)\Big]\Big\}\\
&+\Big(y\leftrightarrow y',\kappa\leftrightarrow\kappa'\Big). \qquad (83)
\end{aligned}
$$

In this expression it is assumed that the derivative fields Φ_0, Π, Ψ and Π_0, apart from the argument explicitly shown, depend on their 'own' rapidity y and transverse point b and also on rapidity y' and external momentum κ', which enter in their definitions (50)–(54). Due to property (67) the last two terms in (83) are obtained from the first two ones by interchanging the target and projectile. Graphical illustration of $I_2^{(\gamma,nf)}$ is presented in Fig. 2, which shows diagrams corresponding to the six terms in (83). The notation is as in Fig. 1. To the diagrams shown in Fig. 2 one has to add the diagrams that are obtained by the interchange of the target and projectile corresponding to the last two terms in (83) and also diagrams from the interchange $y\leftrightarrow y'$, $\kappa\leftrightarrow\kappa'$.

5 Conclusions

We have derived expressions for the single and double inclusive cross sections in nucleus–nucleus collisions in the framework of the perturbative QCD with interacting BFKL pomerons, in the quasi-classical approximation (without loops). The cross sections include terms with emissions both from the pomerons and from the triple pomeron vertex. The obtained single inclusive cross sections are simple. As expected from the AGK rules they reduce to emissions from the central pomeron in the convolution of two sets of fan diagrams connecting it with the projectile and target and from the two neighboring vertices by which this pomeron splits into fans. This expected form of the single inclusive cross sections was used in the calculations in [7].

In contrast, the expressions for the double inclusive cross sections are much more complicated. A part is just the product of two single inclusive cross sections corresponding to independent emissions of two jets from different points in the nuclear overlap transverse space. The other part, however, includes all sorts of rescattering corrections and needs summation of diagrams of a structure different from fans. It also includes emissions from the triple pomeron vertex having different forms for different cuts passing through the vertex. We have set up evolution equations that allow one to find the rescattering corrections to the double inclusive cross sections. Different forms of emission from the vertex have been found in [8]. So in principle the theoretical basis for the calculation of the double inclusive cross section is completed. However, practical computations along these lines seem to be quite complicated and apparently present a separate (and formidable) task.

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Appendix A: The unitarity equation with cut pomerons

The derivation with BFKL pomerons follows that in [4] for local pomerons. The starting point is the equation that tells that the sum of all Green functions in the cut pomeron theory is equal to the sum of all Green function in the original theory, provided the couplings of the fields to themselves and external sources are chosen in accordance with the AGK rules. It is written as

$$
\begin{aligned}
&\sum_{n,m}\frac{g^n}{n!}G^{(n,m)}\frac{f^m}{m!}\\
&=\sum_{n_i,m_j}{}'\,\frac{\mathrm{i}g^n(\sqrt{2})^{n_c-1}}{n_+!n_-!n_c!}G^{(n_i,m_j)}\frac{\mathrm{i}f^m(\sqrt{2})^{m_c-1}}{m_+!m_-!m_c!}. \qquad (A.1)
\end{aligned}
$$

Here $G^{(n,m)}$ are the Green functions in the original theory with n incoming and m outgoing pomerons and $G^{(n_i,m_j)}$ are the Green functions in the cut pomeron theory, where $i,j=+,-,c$, n_i is the number of incoming pomerons of type i, $n=n_++n_-+n_c$ and m_j is the number of outgoing pomerons of type j, $m=m_++m_-+m_c$. The symbol $\sum'$ means that one should drop terms with $n_\pm=n_c=0$ and $m_\pm=m_c=0$, which are absent in the cut theory. The Green functions are assumed to be operators acting in the rapidity-transverse momentum space, convoluted with the external sources given by functions g and f.

One then uses the obvious properties

$$G^{(n_\pm=n,n_\mp=n_c=0;m_\pm=m,m_\mp=m_c=0)}=G^{(n,m)}\,, \quad (A.2)$$

$$G^{(n_\pm=n,n_\mp=n_c=0;m_\mp=m,m_\pm=m_c=0)}=0\,, \qquad (A.3)$$

to consider the sum (A.1) without restrictions on the summations over n_i and m_j and multiplied by 2. One has

$$\sum_{n_i,m_j} \frac{\mathrm{i}g^n(\sqrt{2})^{n_\mathrm{c}}}{n_+!n_-!n_\mathrm{c}!} G^{(n_i,m_j)} \frac{\mathrm{i}f^m(\sqrt{2})^{m_\mathrm{c}}}{m_+!m_-!m_\mathrm{c}!}$$
$$= 2\sum_{n,m>0}\sum_{n_i,m_j} 2\frac{\mathrm{i}g^n(\sqrt{2})^{n_\mathrm{c}-1}}{n_+!n_-!n_\mathrm{c}!} G^{(n_i,m_j)} \frac{\mathrm{i}f^m(\sqrt{2})^{m_\mathrm{c}-1}}{m_+!m_-!m_\mathrm{c}!}$$
$$+ 2\sum_{n,m>0} \frac{g^n}{n!} G^{n,m)} \frac{f^m}{m!} + 1\,, \tag{A.4}$$

where in the first sum of the second equality $n = n_+ + n_- + n_\mathrm{c}$ and $m = m_+ + m_- + m_\mathrm{c}$. However, according to (A.1) the first term is exactly equal to the second one with opposite sign. So we are left with

$$\sum_{n_i,m_j} \frac{\mathrm{i}g^n(\sqrt{2})^{n_\mathrm{c}}}{n_+!n_-!n_\mathrm{c}!} G^{(n_i,m_j)} \frac{\mathrm{i}f^m(\sqrt{2})^{m_\mathrm{c}}}{m_+!m_-!m_\mathrm{c}!} = 1\,. \tag{A.5}$$

The sum in (A.5) is just the total S matrix in the cut pomeron theory, so that the equality (A.5) means that action $A_\mathrm{c}(\xi = 0) = 0$.

In the theory with the action $A_\mathrm{c}(\xi)$ the amplitude $T_\mathrm{c}(\xi)$ is

$$T_\mathrm{c}(\xi) = \frac{1}{2}\sum_{n_i,m_j}{}' \frac{g^n(\sqrt{2})^{n_\mathrm{c}}}{n_+!n_-!n_\mathrm{c}!} G_\mathrm{c}^{(n_i,m_j)}(\xi) \frac{f^m(\sqrt{2})^{m_\mathrm{c}}}{m_+!m_-!m_\mathrm{c}!}\,. \tag{A.6}$$

Adding and subtracting terms with $n_+ = n_\mathrm{c} = 0$ and $m_- = m_\mathrm{c} = 0$ and using properties (A.2) and (A.3) we find

$$T_\mathrm{c}(\xi) = \frac{1}{2}\sum_{n_i,m_j} \frac{g^n(\sqrt{2})^{n_\mathrm{c}}}{n_+!n_-!n_\mathrm{c}!} G_\mathrm{c}^{(n_i,m_j)}(\xi) \frac{f^m(\sqrt{2})^{m_\mathrm{c}}}{m_+!m_-!m_\mathrm{c}!}$$
$$- \sum_{m,n} \frac{g^n}{n!} G^{n,m)} \frac{f^m}{m!} + \frac{1}{2}\,, \tag{A.7}$$

where again in the first sum $n = n_+ + n_- + n_\mathrm{c}$ and $m = m_+ + m_- + m_\mathrm{c}$. The first sum is just one half of $\mathrm{e}^{-A_\mathrm{c}(\xi)}$, while the second is e^{-A} in the original theory, so that (A.7) leads to (21).

Appendix B: Cut vertex functions γ_d, γ and γ_c

The three cut vertex functions γ_d, γ and γ_c which enter (70) can be most conveniently written in coordinate space as functions Γ_d, Γ and Γ_c acting on non-amputated pomeron functions according to (74). To simplify notation we introduce a vector

$$\mathbf{h}(r_1, r_2) = \frac{\mathbf{r}_1}{r_1^2} - \frac{\mathbf{r}_1 + \mathbf{r}_2}{(r_1 + r_2)^2}\,. \tag{B.1}$$

From the results obtained in [8]) we find

$$\Gamma_d(r_1|r_2, r_3) =$$
$$\frac{2\alpha_\mathrm{s}N_\mathrm{c}}{(2\pi)^2}\nabla_1^4\Big\{ \mathrm{e}^{\mathrm{i}\kappa(r_2-r_3)}\mathbf{h}(r_2,r_1)\mathbf{h}(r_3,r_1)$$
$$- \mathrm{e}^{\mathrm{i}\kappa(r_2-r_3+r_1)}\mathbf{h}(r_2,r_1)\mathbf{h}(r_3,-r_1)$$
$$+ 4\mathrm{i}\pi\,\mathrm{e}^{\mathrm{i}\kappa r_2}\left(1-\mathrm{e}^{\mathrm{i}\kappa r_1}\right)\frac{\kappa}{\kappa^2}\mathbf{h}(r_2,r_1)\delta^2(r_3-r_1)$$
$$+ \frac{4\pi^2}{\kappa^2}(1-\mathrm{e}^{\mathrm{i}\kappa r_1})\delta^2(r_2-r_1)\delta^2(r_3-r_1)$$
$$- \frac{4\pi^2}{\kappa^2}\nabla^{-2}\,\mathrm{e}^{\mathrm{i}\kappa r_1}\nabla_1^2\delta^2(r_2-r_1\delta^2(r_3-r_1))\Big\}, \tag{B.2}$$

$$\Gamma(r_1|r_2, r_3) =$$
$$\frac{\alpha_\mathrm{s}N_\mathrm{c}}{2(2\pi)^2}\nabla_1^4\Big\{ \mathrm{e}^{\mathrm{i}\kappa(r_2-r_3)}\mathbf{h}(r_2,r_1)\mathbf{h}(r_3,r_1)$$
$$- 2\,\mathrm{e}^{\mathrm{i}\kappa(r_2-r_3+r_1)}\mathbf{h}(r_2,r_1)\mathbf{h}(r_3,-r_1)$$
$$- 6\mathrm{i}\pi i\,\mathrm{e}^{\mathrm{i}\kappa r_2}\frac{\kappa}{\kappa^2}\mathbf{h}(r_2,r_1)\delta^2(r_3-r_2)$$
$$- 3\mathrm{i}\pi\,\mathrm{e}^{\mathrm{i}\kappa r_2}\left(1-\mathrm{e}^{\mathrm{i}\kappa r_1}\right)\frac{\kappa}{\kappa^2}\mathbf{h}(r_2,r_1)$$
$$\times\delta^2(r_3-r_1-r_2)$$
$$- 3\,\mathrm{e}^{\mathrm{i}\kappa r_3}\mathbf{h}(r_2,r_1)\mathbf{h}(r_2+r_3,r_1)$$
$$+ 2\mathrm{i}\pi\,\mathrm{e}^{\mathrm{i}\kappa r_2}\left(1-2\,\mathrm{e}^{\mathrm{i}\kappa r_1}\right)\frac{\kappa}{\kappa^2}\mathbf{h}(r_2,r_1)$$
$$\times\delta^2(r_3-r_1)$$
$$+ \frac{4\pi^2}{\kappa^2}(1+\mathrm{e}^{\mathrm{i}\kappa r_1})\delta^2(r_2-r_1)\delta^2(r_3-r_1)$$
$$- \frac{16\pi^2}{\kappa^2}\nabla^{-2}\,\mathrm{e}^{\mathrm{i}\kappa r_1}\nabla_1^2\delta^2(r_2-r_1\delta^2(r_3-r_1))\Big\}, \tag{B.3}$$

$$\Gamma_\mathrm{c}(r_1|r_2, r_3) =$$
$$\frac{2\alpha_\mathrm{s}N_\mathrm{c}}{(2\pi)^2}\nabla_1^4\Big\{ - \mathrm{e}^{\mathrm{i}\kappa(r_2-r_3+r_1)}\mathbf{h}(r_2,r_1)\mathbf{h}(r_3,-r_1)$$
$$- 3\,\mathrm{e}^{\mathrm{i}\kappa r_3}\mathbf{h}(r_2,r_1)\mathbf{h}(r_2+r_3,r_1)$$
$$- 3\mathrm{i}\pi\,\mathrm{e}^{\mathrm{i}\kappa r_2}\left(1-\mathrm{e}^{\mathrm{i}\kappa r_1}\right)\frac{\kappa}{\kappa^2}\mathbf{h}(r_2,r_1)$$
$$\times\delta(r_3-r_1-r_2)$$
$$+ 2\mathrm{i}\pi\,\mathrm{e}^{\mathrm{i}\kappa r_2}\frac{\kappa}{\kappa^2}\mathbf{h}(r_2,r_1)\delta(r_3-r_2)$$
$$+ 2\mathrm{i}\pi\,\mathrm{e}^{\mathrm{i}\kappa r_2}\frac{\kappa}{\kappa^2}\mathbf{h}(r_2,r_1)\delta(r_3-r_1)$$
$$- 16\pi^2\nabla_1^{-2}\,\mathrm{e}^{\mathrm{i}\kappa r_1}\delta^2(r_2-r_1\delta^2(r_3-r_1)$$
$$- \frac{8\pi^2}{\kappa^2}\nabla^{-2}\,\mathrm{e}^{\mathrm{i}\kappa r_1}\nabla_1^2\delta^2(r_2-r_1\delta^2(r_3-r_1)\Big\}. \tag{B.4}$$

Due to the symmetry properties of the pomerons, these expressions have to be symmetrized in r_2 and r_3. Also all exponentials have to be substituted for by their real and imaginary parts:

$$\mathrm{e}^{\mathrm{i}kr} \to \cos kr\,, \quad \mathrm{i}\,\mathrm{e}^{\mathrm{i}kr} \to -\sin kr\,, \tag{B.5}$$

which makes the cut vertices real.

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